**DE-BROGLIE MATTER WAVE**

***Wave Particle Dualism***

The concept of wave nature of matter arose from dual character radiation which sometimes behaves as a wave and at other times as a particle. For example, radiation considered as a wave in propagation experiments like interference, diffraction and polarisation. These experiments firmly establish the wave nature of radiation.

On the other hand the phenomenon like photo-electric effect and Compton Effect shows that radiation behaves as a particle- the photon. Thus radiation has a dual characteristic. However, radiation does not exhibit its wave and particle aspect simultaneously. Radiation thus, behaves as a wave transmission and as a particle when it interacts with matter.

In accordance with the theory of relativity Einstein had proved the equivalence of mass and energy i.e. E=mc2.

Similarly, in accordance with the quantum theory of radiation proposed by Max Planck a radiation frequency consist of quanta or photon each of energy h. These and other theoretical considerations lead de-Broglie to prove in 1924 that matter also had a dual (particle as well as wave –like) character. According to him electronns as well as protons which ordinarily behave like particles, under certain conditions, behave like a train of waves. The wavelength of these waves depends upon the momentum which in turn depends upon the mass and velocity of the particles.

The waves associated with a moving particle are called matter waves.

***de-Broglie wave***

In 1924 Luis de-Broglie proposed that *matter possesses wave as well as particle properties*. This wave is called de-Broglie wave.

A photon of light of frequency** has the momentum

The wavelength of a photon is therefore specified by the relation

This wavelength is called de-Broglie wavelength.

de-Broglie suggested that equation (1) is completely general one that applies to material particle as well as photons.

The momentum of a particle of mass *m* and velocity *v* is given by

and its de-Broglie wavelength is accordingly

***Thus, the greater the particle’s momentum, the shorter its wavelength***.

We know,

Again, we know

This is the expression for the Broglie wavelength in terms of energy.

***Wave velocity and group velocity***

The equation of plane progressive wave is given by

Here is called the angular frequency of the wave and *k* is called the wave vector.

Here is called the phase/wave velocity. In other words, is the velocity with which a plane progressive wavefront travels forward. It has a constant phase

Differentiating this equation with respect to t,

The **phase/wave velocity** of a [wave](http://en.wikipedia.org/wiki/Wave) is the rate at which the [phase](http://en.wikipedia.org/wiki/Phase_(waves)) of the wave [propagates in space](http://en.wikipedia.org/wiki/Wave_propagation). This is the [velocity](http://en.wikipedia.org/wiki/Velocity) at which the phase of any one [frequency](http://en.wikipedia.org/wiki/Frequency) component of the wave travels.

Or, equivalently, in terms of the wave's [angular frequency](http://en.wikipedia.org/wiki/Angular_frequency) *ω*, which specifies the number of [oscillations](http://en.wikipedia.org/wiki/Oscillation) per unit of time, and [wave number](http://en.wikipedia.org/wiki/Wavenumber) *k*, which specifies the number of oscillations per unit of space, by

The **group velocity** of a [wave](http://en.wikipedia.org/wiki/Wave) is the [velocity](http://en.wikipedia.org/wiki/Velocity) with which the overall shape of the waves' amplitudes-known as the modulation or [envelope](http://en.wikipedia.org/wiki/Envelope_(waves)) of the wave-propagates through space.

The group velocity vg is defined by the equation:

where ω is the wave's [angular frequency](http://en.wikipedia.org/wiki/Angular_frequency) (usually expressed in [radians per second](http://en.wikipedia.org/wiki/Radians_per_second)), and k is the [angular wave number](http://en.wikipedia.org/wiki/Angular_wavenumber) (usually expressed in radians per meter).

***Relation between wave and group velocity***

Let us consider that the wave group arises from the combination of two waves that have the same amplitude *A* but differ an amount *d* in angular frequency and an amount *dk* in wave number. If *y1* and *y2* are the displacement of the two waves respectively, then

The resultant displacement at any position *x* at any time *t* is given by

Since *d* and *dk* are small compared with ** and *k*

So that equation (3) can be written as

The wave velocity of a [wave](http://en.wikipedia.org/wiki/Wave) is

The group velocity of a [wave](http://en.wikipedia.org/wiki/Wave) is

The angular frequency and wave number of the de-Broglie wave group associated with a moving body of mass *m* moving with the velocity *v* are

The group velocity of de-Broglie wave group associated with a moving body is

Differentiating equation (5) with respect to *v* we get

Again, differentiating equation (6) with respect to *v* we get

From equation (7) we get the group velocity as

Thus, the de-Broglie wave group associated with a moving body travels with the same velocity as the body.

We can also say that *the group velocity is equal to the wave velocity*.

***Mathematical Problems***

***Problem-1:*** *If the de-Broglie wavelength of an electron is equal to . Calculate its kinetic energy.*

***Solution:*** We know

Here,